REGIONAL INFORMATION FOR ECONOMIC, DEMOGRAPHIC & ENERGY ANALYSIS

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POLIS
The Land Use Information and
Transportation System for the
San Francisco Bay Area

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POLIS

THE LAND USE INFORMATION AND TRANSPORTATION SYSTEM FOR THE SAN FRANCISCO BAY AREA

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The Projective Optimization Land use Information System (POLIS) is designed to provide land use, employment, housing and population forecasts for the subregional areas in the San Francisco Bay Area. Employment data is presented by five sectors --agriculture, manufacturing, transportation/FIRE, retail trade and services-- for each of the 107 zones. Additional outputs from the system include commuting flows by mode, auto and transit, shopping trips, while the model can be easily extended to estimate volume of retail sales. The model has been calibrated with 1975 and 1980 data and has been used in the PROJECTIONS '83 series of ABAG's forecasts for the San Francisco Bay Area Economy (ABAG, 1983).

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POLIS allocates households and employment on the basis of several criteria. Residential choice is determined by the travel to work behavior and the availability of housing, whereas the location of the retail sector is affected by the proximity to population centers and the inherent attractiveness of a zone for shopping. The locational patterns of the other sectors are influenced by the accessibility to labor, the existence and size of agglomeration economies and the structural characteristics of each sector. Land use controls are explicitly considered in the model and affect both the availability of housing and the location of economic activity.

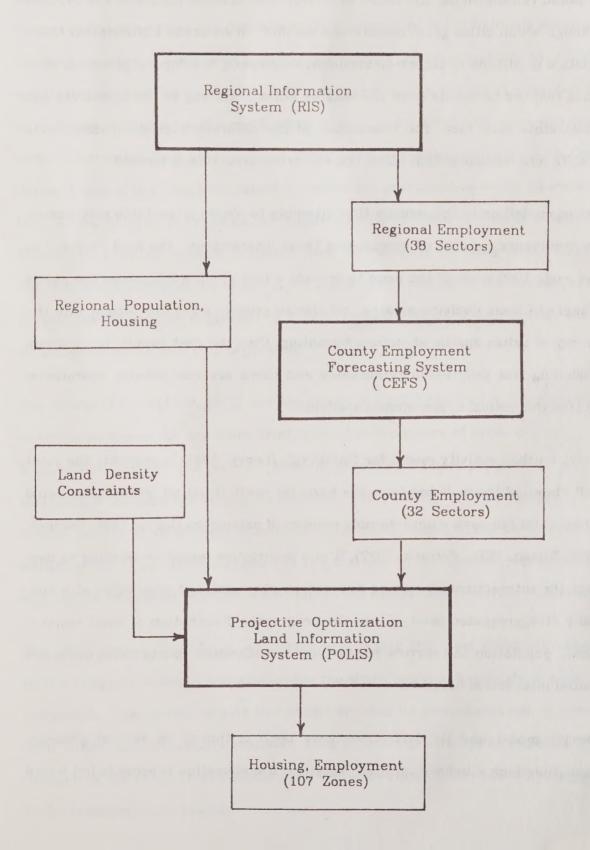
The proposed model is formulated as a nonlinear mathematical programming problem. Hence, the location of residences and housing and trip flows patterns are estimated in a single iteration and are consistent with the constraints of the model and each other. The objective function maximizes the locational surplus, utility, associated with the multimodal travel to work trips, the shopping trips and the agglomeration benefits accruing to employers. The linear constraints reflect the planning constraints, i.e. land use controls, exogenous location of

part of the employment, and accounting identities --total employment to be allocated, origin/destination trip equations etc.--.

POLIS completely replaces BEMOD and PLUM, the employment allocation and land use models for the Bay Area designed in the early '70 's and the major source of all prior ABAG projections. POLIS differs from these models as well as most of the operational Lowry type models in several aspects. It considers in an integrated and internally consistent fashion the location of basic/nonbasic employment and housing. It reproduces the behavior at the micro level and analyzes the location of economic activity in terms of agglomeration economies and proximity to labour, rather than established patterns at the base year. Finally, it lends itself to the testing of issues such as land use controls, scarcity of housing and shifts in the sectoral composition of employment, issues which are expected to be the dominant policy concerns in the Bay Area during the next ten years.

As shown in Figure 1, POLIS is part of the three tiered modeling system of ABAG. The Regional Information System (RIS), a dynamic input-output demographic, economic and energy model estimates regionwide population and employment in 38 sectors (Brady and Yang, 1982). An econometric model, the County Employment Forecasting System (CEFS), allocates the jobs, aggregated to 32 sectors, among the nine counties (Prastacos and Brady, 1983). Since the outputs of both the RIS and CEFS models act as control totals for POLIS, the latter provides a consistent set of subregional projections.

Figure 1
The Modeling System of ABAG



Introduction

The evolution of urban areas is a complex phenomenon full of nonlinear and dynamic relationships. The latter arise from the specific nature of the process through which cities grow, mature and decline. In an urban environment there exists a multitude of actors--households, employers, developers, planners, etc.--that respond to events from the outside world according to the objectives and constraints they face. The interaction of the different decision making units results into feedbacks that affect the way urban structure is formed.

Urban modelling is the science that attempts to obtain a tractable mathematical framework capable of reproducing these interactions. The field emerged in the early 1960's out of the need to provide a tool which would assist the policy makers in their decision making and also to provide a greater insight into the theory of urban spatial structure formation. Over the past twenty years urban modelling has progressed remarkably and there are now several operational and/or theoretical urban models available.

Lowry's urban activity model for Pittsburgh (Lowry, 1964) is probably the most well known of these. It has been the basis for most, if not all, of the operational models and has seen a considerable number of extensions (Garin, 1966; Goldner, 1971; Wilson, 1974; Putman, 1977). It is a descriptive model attempting to capture the interactions occurring between employment and population at a spatially disaggreggated level. Given the geographic distribution of basic employment, population and service employment are allocated among zones using two spatial interaction functions.

Lowry's model and its derivatives have been criticized on several grounds.

First, they lack a behavioral interpretation; the allocation process is not based

on the concepts of utility or profit maximization but merely replicates aggregate trends. The functions used for allocating housing describe base year conditions and fail to address the decision making process of the individuals seeking residences. Second, they tend to disregard many of the economic activity/land use/transportation interactions. Each component of the system is considered in isolation of the rest and a rigid, often ad hoc, cause effect structure and top down information flow are assumed. Third, they are concerned only with the demand side of the land use system. Locators are permitted to locate wherever they desire since no constraints are imposed on the number or type of locators. Fourth, they do not provide a mechanism for handling planning constraints. The iterative solution of the model is not amenable to inequality constraints and despite the procedures suggested by Wilson (1971) the effect of land density constraints can not be easily incorporated.

The efforts of Mills (1967, 1972) and Ingram et. al. (1973) resulted in the creation of urban models which are more firmly rooted in the theory of urban economics. Mills follows Herbert and Stevens (1960) in proposing a linear programming formulation of the land use allocation problem. The difficulty with this approach is that it results in an efficient allocation of land uses which is not necessarily a realistic one. The allocation process is governed by a function representing aggreggate objectives (economic efficiency etc.) rather than individual behavior.

The NBER model (Ingram et. al., 1972) is probably the most elaborate urban model designed to date. It simulates the housing market and consists of a set of submodels. The problems with this model are that its implementation requires data which are not usually available, that it does not attempt to simulate the location of the employment sectors and that only rudimentary attention is paid to the transportation sector.

More recently Coelho and Williams (1978) and Sharpe and Karlquist (1980) have presented a framework which overcomes some of the limitations associated with the Lowry and Mills models. Their models are formulated as mathematical programming problems and have been shown to generate Lowry-type results under certain assumptions. They are based on the concepts of random utility theory which provides a way of reproducing at the aggregate level individual behavior. Both of these models are theoretical and no attempt has been made to date to operationalize them.

This study reports on the design and the empirical estimation of a planning model of this type for the San Francisco region. The proposed model, referred to as the Projective Optimization Land use Information System (POLIS), considers in a unified fashion the location of both basic/service employment and residences and can easily address planning constraints. Although the model is normative it is consistent with the behavior at the micro, individual level. Distribution of trips and modal choice are integrated through logit functions, and the locational patterns of firms are influenced by the existence of agglomeration economies and proximity to labor force.

The modelling effort was guided by three major objectives. First, to produce an operational model which avoids the pittfalls of PLUM -- the Bay Area implementation of the Lowry model -- and encompasses some of the recent advances in the art of urban modelling, second to design a framework for which data are readily available or can be easily obtained and third to provide a tool which is amenable to the testing of planning policy issues.

The set of output variables which can be obtained from POLIS include for each zone:

1. Population

- 2. New housing units
- 3. Employment, disaggreggated to 5 sectors
- 4. Land use characteristics -- vacant land, land consumed --.
- 5. Number of work trips by mode and zone of residence and employment
- 6. Number of shopping trips by zone of residence and location of retail establishments.

Probabilistic Choice Framework for the Lowry Model

The traditional Lowry model is characterized by a sequential decision making process; households first select employment location then look for a residence and finally select the location of other home based activities (shopping, recreation, etc.). Ben Akiva et. al. (1978), Williams and Coelho(1978) and Wilson et. al. (1981) have proposed probabilistic models in which the residential choice is influenced not only by the location of employment but also from the shopping trips location.

Within this framework the utility or surplus U_{ijmk} of living at zone i, working at j, travelling to work by mode m and shopping at k is assumed to be represented by an additive separable function of the form:

$$U_{ijmk} = U_{ij} + U_{m|ij} + U_{k|ijm}$$
 (1)

The first part U_{ij} defines the net utility of living at i and working at j and is affected by the characteristics of zone i as a residential choice, whereas $U_{m|ij}$ is the utility associated with mode m given that i and j were chosen respectively as residential and employment locations. The last part $U_{k|ijm}$ is the net surplus of shopping at location k given that zones i, j and mode m were already selected. Modal choice on the shopping trips could be also introduced. Each of the net utilities can be defined as the difference between the benefits gained and the transportation or interaction costs incurred. All the net utilities are random

variables and therefore the theory of random utility (Domencich and McFadden, 1975; Cochrane, 1975; Williams, 1977) can be readily applied.

Assuming that $U_{k|ijm}$ is independent of j and m, i.e. shopping choice is affected only by the residence location, we have:

$$U_{k|ijm} = u_k^S - c_{ik}^s + \varepsilon_{ik}^S$$
 (2)

$$U_{m|ij} = u_m^W - c_{ijm}^W + \varepsilon_{ijm}^W$$
(3)

$$U_{ii} = u_i^{W} + u_{i*}^{S} - u_{ij*}^{W} + \varepsilon_{ii}$$
(4)

where u_k^S is the trip end utility of shopping at k, u_m^W is the utility associated with mode m, u_i^W is the utility of living at i, $u_{i\bullet}^S$ is a measure of the comparative advantage of shopping from i, $u_{ij\bullet}^W$ is the comparative advantage (or disadvantage) of commuting between i and j c_{ijm}^W is the perceived cost (or disutility) of travelling from i to j with mode m and c_{ik}^S is the travel cost from i to shopping activities at k. Finally, ε_{ik}^S , ε_{ijm}^W and ε_{ij} are random variables representing the variation in choice arising from unobserved factors.

Because U_{ijmk} is additive separable, the maximization of the utility U_{ijmk} is consistent with the choice equation

$$P_{ijmk} = P_{ij} \cdot P_{m|ij} \cdot P_{k|ijm}$$
 (5)

where P_{ijmk} is the probability of selecting zone i as a residential choice, j as an employment location, m as a mode of travel to work and k as a shopping location. P_{ij} is the probability of selecting i and j, $P_{m|ij}$ the conditional probability of choosing m given i and j, and $P_{k|ijm}$ the conditional probability of shopping at k given that i, j and m have been already chosen. If the errors in (2), (3) and (4) are assumed to have a Weibull distribution and to be uncorrelated among the zones, then each of the components of (4) can be expressed as a nested multinomial logit model. That is

$$P_{k|ijm} = \frac{\exp[\beta^{S}(u_{k}^{S} - c_{ik}^{S})]}{\sum_{k} \exp[\beta^{S}(u_{k}^{S} - c_{ik}^{S})]}$$
(6)

and

$$P_{ijmk} = \frac{\exp[\beta^{W}(u_{i}^{W} + u_{i}^{S} - u_{ij}^{W})]}{\sum_{i} \exp[\beta^{W}(u_{i}^{W} + u_{i}^{S} - u_{ij}^{W})]} \cdot \frac{\exp[\lambda(u_{m}^{W} - c_{ijm}^{W})]}{\sum_{m} \exp[\lambda(u_{m}^{W} - c_{ijm}^{W})]} \cdot \frac{\exp[\beta^{S}(u_{k}^{S} - c_{ik}^{S})]}{\sum_{k} \exp[\beta^{S}(u_{k}^{S} - c_{ik}^{S})]}$$
(7)

where β^{W} , λ and β^{S} are parameters related to the standard deviation of the error term in (2), (3) and (4).

Omitting the u_m^W term, i.e. disregarding any utility associated only with mode m and assuming that all the differences between modes are captured by the cost parameter c_{ijm}^W we have,

$$P_{ijmk} = \frac{W_i W_{i^*}^S \exp\left(-\beta^W u_{ij^*}^W\right)}{\sum\limits_{i} W_i W_{i^*}^S \exp\left(-\beta^W u_{ij^*}^W\right)} \cdot \frac{\exp\left(-\lambda c_{ijm}^W\right)}{\sum\limits_{m} \exp\left(-\lambda c_{ijm}^W\right)} \cdot \frac{W_k^S \exp\left(-\beta^S c_{ik}^S\right)}{\sum\limits_{k} W_k^S \exp\left(-\beta^S c_{ik}^S\right)}$$
(8)

where

$$W_{i} = \exp(\beta^{W} u_{i}^{W}) \tag{9}$$

$$W_{i\bullet}^{S} = \exp\left(\beta^{W} u_{i\bullet}^{S}\right) \tag{10}$$

$$W_i^S = \exp(\beta^S u_k^S) \tag{11}$$

The functional form for the trip variables T_{ijm} -number of work trips between i and j by mode m-, and S_{ik} -number of shopping trips between i and k-, is given in terms of total employment E_j and population H_i as,

$$T_{ijm} = E_{j} \cdot P_{ij} \cdot P_{m|ij} =$$

$$= E_{j} \cdot \frac{W_{i}W_{i\bullet}^{S} \exp(-\beta^{W}u_{ij\bullet}^{W})}{\sum_{i} W_{i}W_{i\bullet}^{S} \exp(-\beta^{W}u_{ij\bullet}^{W})} \cdot \frac{\exp(-\lambda c_{ijm}^{W})}{\sum_{m} \exp(-\lambda c_{ijm}^{W})}$$
(12)

$$S_{ik} = H_i \cdot P_{k|ijm} = H_i \cdot \frac{W_k^S \exp(-\beta^S c_{ik}^S)}{\sum_k W_k^S \exp(-\beta^S c_{ik}^S)}$$
(13)

The term u_{ij}^{W} is the mean surplus associated with commuting from i to j. Through (3) it can be defined to be

$$\mathbf{u}_{ij^{\bullet}}^{\mathbf{W}} = -\frac{1}{\lambda} \ln \sum_{m} \exp\left(-\lambda \, \mathbf{c}_{ijm}^{\mathbf{W}}\right) \tag{14}$$

a result already obtained by Williams(1977) for the composite cost by considering the generalized surplus integral of Hotelling (1938). Thus u_{ij}^{W} is the composite cost and can be replaced by the more commonly used symbol \tilde{c}_{ij} .

Equations (12) and (13) define a model which has a similar structure to the results derived from the traditional Lowry model when a multimodal transport system is considered. The attractiveness weights—of the Lowry model are shown to be nonlinear transformations of the trip end utilities. However, since it was assumed that the residential choice is optimized with respect both the work and shopping trips the residential attractiveness factor contains an additional component ($W_{i^*}^{S}$) related to the comparative advantage of shopping from i.

The results obtained above, in addition to providing the Lowry model with a behavioral interpretation, permit the specification and estimation of a consistent measure for the benefits accruing from a specific policy. For example, the locational or consumer's surplus associated with a fixed distribution of { E_j } can be defined as:

$$LS = \sum_{j} E_{j} u_{j}$$
 (15)

where $u_{j^{*}}$, the mean surplus associated with employment at j is computed from (4) as

$$u_{j^*} = \frac{1}{\beta^{W}} \sum_{i} \exp[\beta^{W}(u_i^{W} + u_{i^*}^{S} - c_{ij}^{C})]$$
 (16)

The location surplus can be then written as:

$$LS = \frac{1}{\beta^{W}} \sum_{i} E_{j} \sum_{i} W_{i} W_{i*}^{S} \exp(-\beta^{W} \tilde{c}_{ij})$$
(17)

and used for assessing the benefits arising from a change in the transport system (Neuberger, 1971).

Mathematical Programming Framework

As discussed earlier a major problem with the Lowry model and its derivatives is their inability to handle explicitly planning constraints. In this section we introduce a mathematical programming model which is shown to have a structure similar to that of the probabilistic choice model of the previous section and which can be easily extended to handle density constraints.

Williams (1977) has shown that the total benefits associated with a spatial pattern of trips can be expressed in terms of the variables of a mathematical programming model. The objective function of this model is interpreted as the net utility of a group of individuals and its maximization reproduces the variability existing at the individual level. Coelho and Williams (1978) used this approach to analyze household location when it is influenced by work location and shopping trip behavior. The objective function in that case consists of the sum of two surplus functions, one related to the work trips { T_{ij} } and one to the shopping trips { S_{ij} }. We will extend these results to accommodate a multimodal system (see also Boyce et. al., 1983).

Consider a two sector, service and basic, representation of an urban economy.

Assume that the residential choice is affected by the workplace location, the mode of travel to work and the shopping behavior. The mathematical

programming framework for a spatial interaction model is then given by:

$$\max Z (T_{ijm}, S_{ij}) =$$

$$= \left\{ -\frac{1}{\beta^{w}} \sum_{ijm} T_{ijm} \left[\ln \frac{\sum_{m} T_{ijm}}{W_{i}} - 1 \right] \right\} + \left\{ -\frac{1}{\lambda} \sum_{ijm} T_{ijm} \left[\ln T_{ijm} - 1 \right] - \sum_{ijm} T_{ijm} c_{ijm}^{w} \right\} + \left\{ -\frac{1}{\beta^{s}} \sum_{ij} S_{ij} \left[\frac{\ln S_{ij}}{W_{j}^{S}} - 1 \right] - \sum_{ij} S_{ij} c_{ij}^{S} \right\}$$

$$(18)$$

s.t.

$$\sum_{im} T_{ijm} - d\sum_{i} S_{ij} = 0 \tag{19}$$

$$\sum_{im} T_{ijm} - d' \sum_{i} S_{ij} - d'' E_{j}^{B} = 0$$
 (20)

$$T_{ijm}, S_{ij} \ge 0$$
 (21)

where d, d' and d' are scale parameters and E_j^B is the basic employment in zone j. Constraint (19) ensures that the household population estimated from the work trip matrix is equal to that predicted from the shopping trips while (20) specifies that the total number of work trips to a zone is equal to the total employment (basic and service) in that zone appropriately scaled. The objective function consists of the sum of three surplus functions shown in (18) in brackets. The first two define the locational surplus attached to the residential choice when considering only work trips; the multiple dimensions of the decision process associated with work trips, gives rise to two surplus functions, one for destination and one for modal choice. The last function depicts the locational benefits arising when residential choice is also influenced by the shopping behavior. The parameters β^W , β^S_k and λ convert the utility associated with trip making into monetary units compatible with the transportation costs incurred.

To obtain the number of work trips T_{ijm} consider the Langrangian:

$$L = Z + \sum_{i} \delta_{i} \left(d \sum_{j} S_{ij} - \sum_{jm} T_{ijm} \right) + \sum_{j} \gamma_{j} \left(d' \sum_{i} S_{ij} + d'' E_{j}^{B} - \sum_{im} T_{ijm} \right)$$
(22)

where δ_i and γ_j are the dual variables of constraints (19) and (20). Since for finite β^W , λ and β^S the trip variables T_{ij} , T_{ijm} and S_{ij} are always positive we have from the Kuhn-Tucker conditions;

$$\frac{\partial L}{\partial T_{ijm}} = 0 = -\frac{1}{\beta^{w}} \left[\ln \frac{\sum_{m} T_{ijm}}{W_{i}} \right] - \frac{1}{\lambda} (\ln T_{ijm}) - c_{ijm} - \delta_{i} - \gamma_{j}$$
 (23)

and

$$\frac{\partial L}{\partial S_{ij}} = 0 = -\frac{1}{\beta^{w}} \left[\ln \frac{S_{ij}}{W_{j}^{S}} \right] - c_{ij}^{S} + d \delta_{i} + d' \gamma_{j}$$
 (24)

which can be rewritten in terms of the trip variables as:

$$T_{ijm} = \left(\frac{\sum_{m} T_{ijm}}{W_{i}}\right)^{-\lambda/\beta^{W}} \exp(-\lambda c_{ijm}) \exp[-\lambda(\delta_{i} + \gamma_{j})]$$
 (25)

$$S_{ij} = W_j^{S} \exp\left[\beta^{S} (d\delta_i + d'\gamma_j - c_{ij}^{S})\right]$$
 (26)

Summation of (25) with respect m results to:

$$W_{i} = \frac{\sum_{m} T_{ijm}}{W_{i}} = \left(\frac{\sum_{m} T_{ijm}}{W_{i}}\right)^{-\lambda/\beta^{w}} \sum_{m} \exp(-\lambda c_{ijm}) \exp[-\lambda(\delta_{i} + \gamma_{j}))$$
(27)

or

$$\left(\frac{\sum_{m} T_{ijm}}{W_{i}}\right)^{-\lambda/\beta^{W}} = W_{i}^{\lambda/\lambda+\beta^{W}} \left[\sum_{m} \exp\left(-\lambda c_{ijm}\right)\right]^{-\lambda/\lambda+\beta^{W}} \exp\left[\frac{\lambda^{2}}{\lambda+\beta^{W}}(\delta_{i}+\gamma_{j})\right]$$

Replacing (28) into (25) we get:

$$T_{ijm} = \left[\sum_{m} \exp(-\lambda c_{ijm})\right]^{-\lambda/\lambda + \beta^{W}} \exp(-\lambda c_{ijm}) \exp\left[\frac{-\lambda \beta^{W}}{\lambda + \beta^{W}} (\delta_{i} + \gamma_{j})\right]$$
(29)

Introducing a composite cost function c_{ij} with form similar to that defined in

the probabilistic choice model we get:

$$\exp(-\lambda c_{ij}) = \sum_{m} \exp(-\lambda c_{ijm})$$
(30)

and (29) can be rewritten after some manipulation as,

$$T_{ijm} = W_{i}' \exp \left[\beta' \left(\delta_{i} + \gamma_{j} \right) \right] \exp \left(-\beta' c_{ij} \right) \frac{\exp \left(-\lambda c_{ijm} \right)}{\sum_{m} \exp \left(-\lambda c_{ijm} \right)}$$
(31)

where β' is defined to be equal to $\frac{\beta^{W}\lambda}{\beta^{W}+\lambda}$.

Invoking the constraints for doubly constrained models, the balancing factors A_i^W , B_j^W , A_i^S and B_j^S can be defined as:

$$A_i^{W} = \left[\sum_j B_j^{W} E_j \exp\left(-\beta' \hat{c}_{ij}\right) \right]^{-1}$$
(32)

$$\mathbf{B}_{\mathbf{j}}^{\mathbf{W}} = \left[\sum_{i} \mathbf{A}_{i}^{\mathbf{W}} \mathbf{H}_{i} \exp\left(-\boldsymbol{\beta}' \hat{\mathbf{c}}_{ij}\right) \right]^{-1}$$
(33)

$$A_i^{S} = \left[\sum_j B_j^{S} E_j^{S} \exp\left(-\beta' \hat{c}_{ij}\right) \right]^{-1}$$
(34)

$$B_j^{S} = \left[\sum_i A_i^{S} H_i \exp\left(-\beta' \hat{c}_{ij}\right) \right]^{-1}$$
(35)

and the trips variables expressed as:

$$T_{ijm} = A_i^{W} H_i B_j^{W} E_j \exp(-\beta' \hat{c}_{ij}) \frac{\exp(-\lambda c_{ijm})}{\sum_{m} \exp(-\lambda c_{ijm})}$$
(36)

$$S_{ij} = A_i^S H_i B_j^S E_j^S \exp(-\beta^S c_{ij}^S)$$
(37)

It can be easily demonstrated that the trip patterns (36) and (37) are of the same type with those obtained in the probabilistic model discussed earlier. Thus, the mathematical programming framework, although it has an aggregate nature, results in expressions at the micro level which are consistent with the theory of behavior at that level. Additionally, because of its optimization

nature, it can easily handle planning constraints. The objective function is concave separable subject to linear constraints and the model can be estimated by several available methods.

The dual program associated with (18)-(21) is unconstrained and is given by

$$\min L\left(\delta_{i}, \gamma_{j}\right) = \frac{1}{\beta'} \left\{ \sum_{ijm} W_{i}' \exp\left[-\beta'\left(\delta_{i} + \gamma_{j} + \widetilde{c}_{ij}\right)\right] \frac{\exp\left(-\lambda c_{ijm}\right)}{\sum_{m} \exp\left(-\lambda c_{ijm}\right)} \right\} + \frac{1}{\beta''} \left\{ \sum_{ij} W_{j}' \exp\left(\beta'' \left(d \delta_{i} + d' \gamma_{j} - c_{ij}''\right)\right] \right\} + \sum_{j} d'' \gamma_{j} E_{j}^{B}$$

$$(38)$$

The objective function represents the total locational surplus accruing to locators and suppliers of housing opportunities. The vector of shadow prices γ is the consumer surplus of the individuals seeking a house, while δ can be interpreted as the location rent on housing. As Coelho and Williams(1978) point out estimation of the planning model must be carried out by solving the dual rather than the primal problem, since the number of variables in the dual is significantly smaller.

Location of Employment

In the model considered above only the retail sector's location is determined endogenously. It was assumed that the location of the basic sectors is esimated outside of the model. With the exception to the work of Mackett (1980), this is a characteristic of all the land use models; basic employment location is predicted through a set of indexes and then used to drive the residential/shopping choice model(s). Such an approach assumes that changes in housing location patterns will not influence the location of industries.

Although this approach may be valid for the traditional heavy manufacturing sectors (oil,chemicals,transportation equipment) it may lead into erroneous results when considering the intraurban location of "footloose" industries which are highly dependent on access to labour markets. Electronics, Research and Development and several non retail services -- FIRE, part of Business and Personal Services -- are typical examples of sectors where availability of qualified labor force and proximity to competitors are the major considerations for locational decisions. It must be noted that employment in these sectors represents more than 50% of the non-retail employment af the Bay Area and is expected to grow at a very high rate.

To integrate the location for all employment in the mathematical programming framework the factors influencing the locational decisions of the firms in the basic industries should be added to the criterion function. The most significant of these are: access to labor, access to markets, proximity to suppliers, agglomeration economies and land availability.

The access to labor factor is already part of the model since the objective function represents the employee's consumer surplus with respect to their trip making and residential choices; therefore the accessibility of employment supply to demand is maximized. Proximity to markets and supliers is of particular importance to retailers and industries with low value added per unit of weight ratio, but should not be a major consideration for firms producing for external markets or industries in the finance sector. For high tech type firms proximity to competitors/suppliers is desirable since it provides access to specialized labor.

Agglomeration economies arise form the location of firms adjacent to each other. They include: access to sources of capital, labor market ecomonies,

communication and information economies, access to specialized business services, superior training facilities, attractive environment for innovation and other not easily quantifiable parameters. In the Bay Area they are highly significant and are the major cause for the rapid growth of the high technology industry in the Silicon Valley and the heavy concentration of financial and business services in the city of San Francisco. Unfortunately, they can not be estimated directly and must be defined in terms of surrogate variables.

Agglomeration economies can be integrated into a land use model by separating them between those occurring at the macro level and those exhibited at the zonal level. The first ones consist of the causal relationships that link activities to each other. They represent comparative advantages and profitabilities arising from the existing structure of production and consumption. The intersectoral relationships shown by the Input-Output table is one manifestation of these. The propensity of certain industries to locate in specific areas is another.

They can be incorporated in a mathematical programming model through the addition of equations in the constraint set linking employment in a particular sector to the size of the employment (lagged or current) in other related sectors and counties. These spatial sectoral equations can be defined similar to White and Hewings (1982) or Prastacos and Brady (1983) and take the form:

$$E^{k} = f(E_{t-1}^{k}, \sum_{q \in Q} E^{q}, E^{q})$$
(39)

where Q is the set of sectors with which strong economic relationships exist, t-1 indicates a lagged value and the superscript * denotes regional variables. The choice of spatial level in (39) is governed by the availability of information. In order to capture long term trends the coefficients of (39) should be estimated from time series and not cross sectional data. SMSAs, counties or large cities

although they do not necessarily conform to economic boundaries, are appropriate levels for which long data series are usually available. In the proposed model for the San Francisco Area spatial sectoral equations similar to (39) are introduced for each county.

The agglomeration economies at the zonal level arise from the inherent attractiveness of certain zones. They are directly related to particular zonal characteristics and are sensitive to changes in these. Relative cost/availability of land and accessibility to other employment centers are probably the two major factors that render some zones more attractive than others. Using microeconomics principles they can be interpreted as the marginal benefits accruing to locators of employment opportunities.

To incorporate them in the modelling framework assume that there exists a function $f(\cdot)$ representing the agglomeration potential of every zone. Then, the component $f(\cdot)E$ could be introduced in (18) and the objective function be interpreted as the maximization of employer's and employee's surplus. The function $f(\cdot)$ can be an index or a linear/nonlinear transformation of zonal characteristics.

The Land Use Information System for the Bay Area

POLIS allocates employment and households on the basis of the criteria elaborated earlier. Residential choice is related to the travel to work and shopping behavior, while retail activity is located with respect to residences and a factor representing the attractiveness of a zone for shopping. Manufacturing and other basic industries locational patterns are influenced by the accessibility to labor, the existence of agglomeration economies and the existing structure of production.

The model is not dynamic, but it can be classified as quasi-dynamic since it simulates the change between two states. At each time period only the net increase in employment opportunities/households is allocated and relocation of base year jobs is handled by appropriately increasing the number of jobs to be distributed. Thus, the problem of creating an instant metropolis, for which the Lowry model has been criticized, is avoided. The total number of jobs and households to be allocated are given exogenously; hence, the results of the model are consistent with those derived from regional economic models.

In describing the mathematical framework of the system it is assumed that there are N zones and K economic sectors. The K sectors are partitioned into two sets, the basic sectors K_{bas} and the retail or local services sectors K_{loc} . For variables with spatial and sectoral components subscripts identify zones and superscripts represent sectors. Base year values and control totals are denoted respectively with a superscript o and a bar.

The complete model is written:

max Z
$$(T_{iim}, S_{ii}^k, \Delta E_i^n, \Delta H_i) =$$

$$= -\frac{1}{\beta^{w}} \sum_{ijm} T_{ijm} \left(\ln \frac{\sum_{ijm}^{T_{ijm}}}{W_{i}} - 1 \right) - \frac{1}{\lambda} \sum_{ijm} T_{ijm} \left(\ln T_{ijm} - 1 \right) - \sum_{ijm} T_{ijm} c_{ijm} - 1$$

$$-\sum_{\mathbf{k}\in\mathbf{K}_{loc}}\frac{1}{\beta_{\mathbf{k}}^{\mathbf{S}}}\sum_{ij}S_{ij}^{\mathbf{k}}\left(\ln\frac{S_{ij}^{\mathbf{k}}}{W_{i}^{\mathbf{k}}}-1\right)-\sum_{ij\mathbf{k}}S_{ij}^{\mathbf{k}}c_{ij}+\sum_{i,n\in\mathbf{K}_{loc}}(f_{i}^{n})^{\alpha^{n}}\Delta E_{i}^{n}$$
(40)

subject to

$$\sum_{jm} T_{ijm} - a_i (H_i^o + \Delta H_i) = 0$$
(41)

$$\sum_{im} T_{ijm} - \sum_{n} b_j^n (E_j^{no} + \Delta E_j^n) = 0$$
(42)

$$\sum_{i} S_{ij}^{k} - e_{i}^{k} (H_{i}^{o} + \Delta H_{i}) = 0$$
 (43)

$$\sum_{i} S_{ij}^{k} - h^{k} (E_{j}^{ko} + \Delta E_{j}^{k}) = 0$$
 (44)

$$\sum_{n} d^{n} \Delta E_{j}^{n} \leq \bar{L}_{j} \tag{45}$$

$$\Delta \overline{H}_{i,lb} \leq \Delta H_i \leq \overline{V}_i$$
 (46)

$$\sum_{j} \Delta E_{j}^{n} - \overline{E}_{*}^{n} = 0 \tag{47}$$

$$\sum_{i} \Delta H_{i} - \overline{H}_{*} = 0 \tag{48}$$

$$\sum_{j \in P_c} \Delta E_j^n - \sum_{j \in P_c} \sum_{q \in Q} l^q \Delta E_j^q - y_c^n = 0$$

$$\tag{49}$$

$$\Delta \overline{E}_{j,lb}^{n} \leq \Delta E_{j}^{n} \leq \Delta \overline{E}_{j,ub}^{n}$$
 (50)

$$T_{ijm}, S_{ij}^{k}, \Delta H_i, \Delta E_j^n \geq 0$$
 (51)

where ΔH_i is the number of new households in i, ΔE_j^n is the number of additional jobs in sector n to be located at j, \overline{L}^j is the area of land available for job location, \overline{V}_i is the land available for housing at i and the rest of the variables take the definitions provided earlier. P_c is the set of zones belonging in county c.

The objective of the model maximizes the locational surplus associated with the multimodal travel to work trips, the shopping trips and the agglomeration benefits accruing to employers. Cost components are not included since the marginal cost of establishing a household or setting up a firm can not be estimated with any degree of accuracy. Equations (41) and (42) are the origin destination constraints for work trips and (43)-(44) fullfill the same function for the shopping trips. The coefficients e_i^k represent mean expenditure per household of zone i for goods of sector k, hence, the flow variables S_{ij}^k are interpreted as the volume of sales of goods of type k in zone j to residents of i.

Constraints (45) and (46) are the land use constraints for employment and

housing. Constraints (50) and the first part of (46) are introduced for some zones and sectors to account for exogenous location of a portion of the employment (for example government) or other zoning considerations which are not captured in (45). Equations (47) and (48) define the total number of jobs/households to be allocated. Finally, linear constraints similar to (49) are introduced for each basic sector and county to reproduce established sectoral-spatial patterns and agglomeration economies at the macro level; they will be further discussed in the next section.

The dual problem is written:

$$\begin{split} & \min \ L \ \left(\ \kappa_{i} \ , \ \vartheta_{j} \ , \ \eta_{i}^{k} \ , \ \mu_{j}^{k} \ , \ \nu_{j} \ , \ \rho_{i} \ , \ \xi^{n} \ , \ \tau \ , \ \varphi_{c}^{n} \ \right) = \\ & = \ \frac{1}{\beta'} \left\{ \sum_{ijm} W_{i}' \exp \left[-\beta' \left(\kappa_{i} + \vartheta_{j} + \widetilde{c}_{ij} \right) \right] \frac{\exp \left(-\lambda c_{ijm} \right)}{\sum_{m} \exp \left(-\lambda c_{ijm} \right)} \right\} + \\ & + \sum_{k} \frac{1}{\beta^{S}} \left\{ \sum_{ij} W_{j}^{S} \exp \left(\beta^{S} \left(\ \eta_{i}^{k} + \ \vartheta_{j} - c_{ij}^{S} \right) \right] \right\} + \sum_{i} a_{i} \kappa_{i} H_{i}^{o} + \sum_{nj} b_{j}^{n} \vartheta_{j} E_{j}^{no} + \\ & + \sum_{ik} e_{i}^{k} \eta_{i}^{k} H_{i}^{o} + \sum_{kj} h^{k} \mu_{j}^{k} E_{j}^{ko} + \sum_{j} \nu_{j} \overline{L}_{j} + \sum_{i} \rho_{i} \overline{V}_{i} + \sum_{n} \xi^{n} \overline{E}_{i}^{n} + \tau \overline{H}_{i} + \sum_{cn} y_{c}^{n} \varphi_{c}^{n} \right. \end{split}$$

such that

$$b_j^n \vartheta_j - d^n \nu_j - \xi^n - \varphi_c^n + \sum_{\alpha} l^n \varphi_c^q \ge (f_i^n)^{\alpha^n}$$
 (53)

$$b_j^k \vartheta_j + h^k \mu_j^k - d^k \nu_j - \xi^k + \sum_q l^k \varphi_c^q \leq 0$$
 (54)

$$a_i \kappa_i + e_i^k \eta_i^k - \rho_i - \tau \le 0 \tag{55}$$

where κ_i , ϑ_j , η_i^k , μ_j^k , ν_j , ρ_i , ξ^n , τ , φ_c^n are the dual variables of constraints (41)-(49).

There are two different ways of solving the proposed model. The first approach

is to solve the dual problem (52)-(55) using a nonlinear programming algorithm. The dual consists of a set of linear constraints and a concave function whose gradient can be readily estimated analytically at any feasible point, hence, any of the standard nonlinear programming algorithms (Conjugate Gradient, Convex Simplex, Quasi-Newton) can be employed. At optimality, the values of the dual variables uniquely define the primal ones and the optimal set of trip flows and population/employment allocation can be estimated as functions of the optimal dual variables (eq. (29), (26) and (42)). It must be stressed that, although the primal problem can be solved directly through a nonlinear programming algorithm --convex objective function subject to linear constraints--, the solution of the dual is preferable because the latter has a significantly smaller number of variables and constraints.

The second approach exploits the structure of the primal. Since the nonlinearities arise from the trip variables and since the problem reduces to a trip distribution model when the housing and employment variables are fixed, a partitioning or decomposition algorithm can be applied. Benders' Partitioning Algorithm (BPA) (Benders, 1962; Geoffrion, 1972) is a natural candidate for such an application. It is an iterative algorithm which guarantees convergence to the optimal solution in a finite number of steps and which can provide a heuristic solution if computer time limitations force an early termination of the solution procedure.

When applying BPA the model is partitioned into two problems, a master problem which is a linear programming model estimating the location of housing (ΔH) and employment (ΔE) and a subproblem which is a combined distribution-modal split problem computing number of work and shopping trips. In the subproblem the trip origins and destinations take the values of the just computed ΔH and ΔE respectively. The solution algorithm consists of successions.

sively solving the master problem and the subproblem until convergence is reached. The dual of the distribution-modal split model gives rise to a constraint related to trip costs and accessibilities which is then added to the master problem and the procedure is repeated until an acceptable level of convergence is achieved.

The complete solution algorithm is outlined below and also shown in Figure 2. A more complete description of the algorithm and its convergence properties can be found in Prastacos (1984). An index is added as a superscript on the left side of a variable to denote the iteration number.

- Step 0. Select a convergence parameter $\varepsilon \ge 0$. Set the upper bound $UB = +\infty$ and the lower bound $LB = -\infty$. Set the number of iterations N = 0. If a feasible set of vectors ${}^1\Delta H$, ${}^1\Delta E$ exists go to step 1; otherwise go to step 2.
- Step 1. Increase N by one and compute the following combined distribution-modal split problem for fixed values of $^{N}\Delta H$ and $^{N}\Delta E$.

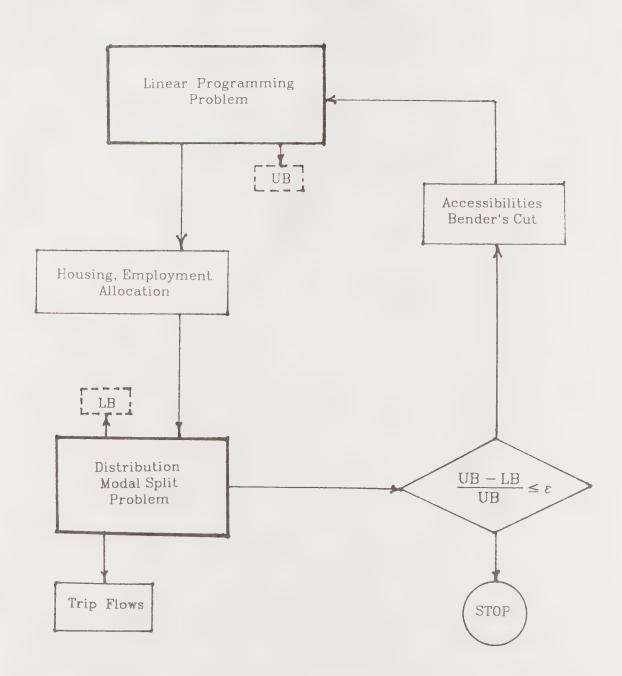
$$\max W = -\frac{1}{\beta^{w}} \sum_{ijm} T_{ijm} \left(\ln \frac{\sum_{m} T_{ijm}}{W_{i}} - 1 \right) - \frac{1}{\lambda} \sum_{ijm} T_{ijm} \left(\ln T_{ijm} - 1 \right) - \frac{1}{\lambda} \sum_{ijm} T_{ijm} \left(\ln T_{ijm} - 1 \right) - \sum_{ijm} T_{ijm} c_{ijm} - \sum_{k \in K_{loc}} \frac{1}{\beta_{k}^{S}} \sum_{ij} S_{ij}^{k} \left(\ln \frac{S_{ij}^{k}}{W_{j}^{k}} - 1 \right) - \sum_{ijk} S_{ij}^{k} c_{ij}$$
(56)

subject to the constraints (41) through (44). The two distribution problems, one for T_{ijm} and the other for S_{ij}^k , can be estimated independently of ech other.

Denote the optimal value $^{N}W.$ Set $LB=^{N}F+^{N}Z_{o}-^{N}W$ if $LB\geq ^{N}F+^{N}Z_{o}-^{N}W\;.$

Test for convergence: If $\frac{UB-LB}{UB} \le \varepsilon$ the procedure has converged. Otherwise estimate the optimal dual variables κ_i , ϑ_j , η_i^k and μ_i^k

Figure 2
The Solution Algorithm



associated with constraints (41) - (44) of the distribution problems and go to step 2.

Step 2. Estimate the following linear programming problem.

$$\max F = Z_o + \sum_{i,n \in K_{bas}} (f_i^n)^{\alpha^n} \Delta E_i^n$$
 (57)

subject to:

$$Z_{o} \leq \sum_{i} (\kappa_{i} a_{i} + \eta_{i}^{k} e_{i}^{k}) \Delta H_{i} + \sum_{j,n} \vartheta_{j} b_{j}^{n} \Delta E_{j}^{n} + \sum_{j,k \in K_{loc}} \mu_{j}^{k} h^{k} \Delta E_{j}^{k} + C$$
 (58)

and constraints (45) through (51).

C is a constant obtained from the dual of the distribution problems. Let $^{N+1}\Delta H$, $^{N+1}\Delta E$ and ^{N+1}F denote respectively the optimal variables and value of this problem. Set $UB=^{N+1}F$. Go to step 1.

At the time this modelling effort was carried out we did not have access to a nonlinear programming computer package. Thus, the Bender's Partitioning Algorithm was exclusively used for calibrating and estimating POLIS. A special computer program was written to implement the algorithm and carry out the solution iterations. The linear programming master problem is estimated using XMP (Marsten, 1981), a hierarchically structured library of subroutines for linear programming, written and distributed by Professor Ray Marsten of the Department of Informations Systems at the University of Arizona. The combined distribution-modal split problem is computed using a standard entropy maximizing algorithm.

Empirical Estimation of the Model

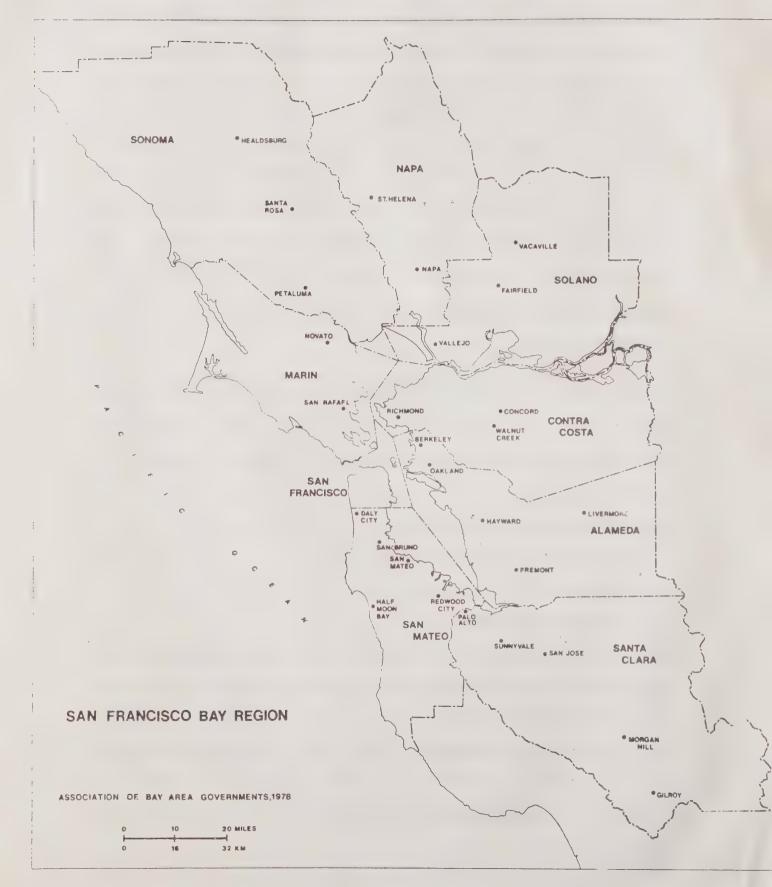
The complete land use transportation model was applied to the San Francisco Bay Area (Figure 3) using aggregate data. The empirical estimation relied only on already available information, since lack of resources did not permit the undertaking of any major data collection effort, i.e. household surveys, special tabulations. Our experience with the implementation of the model indicates that models of this type can be successfully estimated only when Census information are available.

At the time this research took place the complete data sets for the 1980 Census were not yet released. As a result, the model was calibrated using data from two different time periods 1975 and 1980. Information on the spatial distribution of jobs, housing and population were available for both 1975 - in house data base form a previous modelling effort - and 1980 - but trip table and trip cost data were available only for the year 1975. It is hoped that with the recent release of Urban Transportation Planning Package (UTPP) from the 1980 Census this discrepancy will be overcomed.

The transportation data were obtained from the Metropolitan Transportation Commission (MTC), the transportation planning agency for the Bay Area. They included 1975 trip generation and attraction rates, detailed (440 zones) travel to work trip tables by mode, peak-hour level of service characteristics (travel time, distance) and values of time for the different modes. The trip table was 'synthetic', in the sense that it was not produced from primary information but was obtained through MTC's disaggregate travel demand modelling system (CSI, 1980) and validated to replicate observed travel patterns at an aggregate level - 30 zones- (Kollo et. al., 1981). Information on shopping trips was more scarce and consisted of an aggregate trip flows matrix and a detailed non-peak hour

Figure 3

The San Francisco Bay Area



level of service table.

A special emphasis was placed in developing an uptodate accurate estimate of the amount of developed land and the supply of land. The Local Development Policy Survey Update, conducted in stages during 1981-82 by ABAG staff with the cooperation of local government agencies, revised the land use consumption data from the original 1976 ABAG Local Policy Survey. Development activity that occurred over the 1975-1980 period and post 1980 development potential, including both new development on vacant land and redevelopment, were assessed for each jurisdiction and merged with the earlier information. The resulting data base consisted of detailed land use profiles --developed land by type, vacant land and the potential for new jobs or residences -- for each census tract of the Bay Area.

For the purpose of the model the nine counties of the Bay Area were divided into 107 zones, each of the zones representing an aggregation of census tracts. Two modes, auto and transit, and four employment sectors were considered. The sectors and their SIC correspondence were:

- 1. Manufacturing (SIC 20-39, 50-51)
- 2. Transportation, FIRE (SIC 40-49, 60-67)
- 3. Retail Trade (SIC 52-59)
- 4. Services (SIC 70-89)

This sectoral aggregation was necessitated in order to reduce the computational difficulties and most important to be able to produce forecasts at a level which is compatible with other studies of ABAG and MTC. It is not the most appropriate one because it puts together sectors with different locational characteristics. For example, Transportation, a sector which is evenly distributed throughout the region, is lumped with FIRE, a sector which is highly concen-

trated in the county of San Francisco. From the four sectors Manufacturing and Transportation/FIRE were assumed to be basic.

In addition to these four sectors POLIS allocates employment in the Agriculture, Mining sector (SIC 01-14). However, since the employment in that sector represents a very small portion of the total regional employment and since its locational patterns are affected by natural resources, future employment levels are determined as a function of base year conditions and land availability rather than proximity to labor supply.

The implementation of the model to the Bay Area consisted of three major tasks: estimation of the attractiveness weights and agglomeration economies function, specification of the spatial sectoral equations for the basic sectors -eq. (49)-, and finally calibration of the complete model.

The attractiveness factors W_i and W_j^k represent the trip end utilities -eq.(9), (11)- and were defined as functions of different surrogate measures of attraction. A major consideration on the definition of these factors was to estimate them in such a way so they can be easily reproduced in future time periods. In a forecasting environment these factors should not retain base year values but be updated every time period to reflect changing locational patterns. Additionally, if the model is to test alternative policies the attractiveness weights must be designed in such a way so that they are sensitive to the issues being tested. These are accomplished by defining them in terms of variables of the model or other variables which can be easily estimated outside the model.

The attraction of a zone for residence was taken to be equal to:

$$W_{i} = V_{i} \cdot (1 + \frac{H_{i}}{H_{i} + V_{i}} \cdot q_{i})$$
 (59)

where H_i and V_i are respectively the already developed and vacant residential land and q_i is the ratio of median household income and median price of housing at zone i. The q_i ratios are normalized and can be interpreted as housing affordability indexes. They have the effect of rendering more attractive zones for which house prices are not high compared to incomes. Although q_i can not be estimated for future time periods, we feel that being a ratio it is more time invariant than other housing characteristics like price or quality.

For the retail and service sectors zonal attractiveness was defined as the product of the available land, the percentage of already developed land and relative accessibility to new households. That is,

$$W_j^k = L_j \cdot (1 + \frac{Y_j}{Y_j + L_j} \cdot \frac{E_j^k}{E_j^*}) \cdot g_j^k$$
 (60)

where Y_j is the total non residential developed land and L_j is the total available land. The ratio E_j^k / E_j^* converts the percent of total developed land into land occupied by firms of sector k. The Hansen type accesibility index g_j^k was introduced to indicate the propensity of local serving sectors to locate near new population centers. It is specified as:

$$g_j^k = \sum_i \Delta H_{i,t-1} + \exp(-\beta_k^S c_{ij})$$
 (61)

It can be shown that this definition is consistent with the rest of the model. The number of new households is lagged one time period to avoid estimation problems.

The absence of any information on the structure of the zonal agglomeration economies proved to be a major hurdle in the estimation process. There are infinite ways that they can be defined and no prior research work has be done in that field. Four different components were identified as the major components of an agglomeration function: size of employment at base year, rate of

employment growth (sectoral and total) and proximity to other fast growing areas. Since the economies existing at the macro, county, level are captured by equation (49) the function f_i^n must reflect relative intracounty differences appropriately normalized to account for the county and regional rate of growth.

On the basis of these principles the function f_i^n was specified as:

$$f_{i}^{n} = \frac{E_{i}^{n}}{E_{co}^{n}} \cdot \frac{\Delta E_{i}^{n} / \Delta E_{co}^{n}}{\Delta E_{co}^{n} / \Delta E_{i}^{n}} \cdot \frac{\Delta E_{i}^{\bullet} / \Delta E_{co}^{\bullet}}{\Delta E_{co}^{\bullet} / \Delta E_{i}^{\bullet}} \cdot \frac{\Delta E_{co}^{n}}{\Delta E_{co}^{\bullet}}$$
(62)

where the subscript co denotes an employment variable for the county in which i belongs. All the ΔE variables are lagged one time period. Available land was not included in (62) since the agglomeration economies of a zone are more related to the existing rather than the future concentrations of employment.

As discussed earlier the spatial sectoral equations are introduced to account for the agglomeration economies existing at the macro level and the structure of production and consumption. They describe the cumulative effect that these have on the location of industries rather than actually analyze the size or the composition of the economies per se. They were specified for each basic sector and county. For the Manufacturing sector they were defined as:

$$E_{co}^{1} = h \left(E_{co,t-1}^{1}, \Delta E_{t}^{1}, E_{co,t}^{4} \right)$$
 (63)

and for Transportation, FIRE

$$E_{co,t}^2 = h' \left(E_{co,t-1}^2, \Delta E_{*,t-1}^2, E_{*,t}^1, E_{co,t}^3 + E_{co,t}^4 \right)$$
 (64)

Subscripts co and * denote county and regional variables, t is the time period and the functions h and h' are assumed to be linear. The employment E at any time period t is equal to the base year employment plus the change ΔE .

The lagged variable is introduced to account for the historic concentration of employment and the change in regional employment is a proxy for the rate of

growth occurring at the regional level. The other variables are included to identify the cause and effect relationships between the different sectors. It was assumed that local services sectors are linked only with other activities in the same county while for basic sectors the linkages cross county boundaries. Thus, the employment variables of the former enter in the right hand side of (63) and (64) with their county values while those of the latter retain their regional values.

Time series on county employment were obtained from the *County Business Patterns* reports for the year 1964-1980 and the equations (63) and (64) were estimated using ordinary least squares techniques. For almost all the counties and both sectors the fit was very good, R² greater than .90 and high significance values.

The use of (63) and (64) for simulating the impact of the macro agglomeration economies may lead to erroneous results when producing long term forecasts. The coefficients of the regression equations can not be assumed to remain constraint. This problem should be recognized when the model is used for forecasting activity levels for post 1985 years and the spatial sectoral equations should be partially relaxed. For the Manufacturing sectors a confidence interval of the form,

$$(1-r) \cdot h(\cdot) \le E_{co}^{1} \le (1+r) \cdot h(\cdot)$$
(65)

could be defined. The variable r may take the values of .05 for year 1990 and .10 for 1995 and 2000. An alternative procedure would be to specify confidence intervals for each coefficient in the regression; however, this would increase the number of constraints and further burden the computational requirements.

Calibration of the Model

Calibration is the process of choosing values for the parameters of a model which produce the best correspondence between the model's results and the observed situation at the base year. For the proposed model the variables to be calibrated are those which convert the utility associated with trip making into monetary units, namely β^{W} , β^{S}_{k} , λ and the exponents of the agglomeration function α^{n} .

The complex structure of POLIS --simultaneous residential/employment location and trip estimation --, and most important the lack of trip data for the year 1980 did not permit the immediate application of any of the established calibration techniques (Batty, 1976) for models of this type. Hence, an alternative calibration methodology was developed. The complete model was calibrated in three stages. The first two stages involved the calibration of the travel parameters, whereas at the last stage the α^n parameters were obtained.

Initially, the model represented by equations (18)-(21) was calibrated on the 1975 trip data. This is a model where only the location of the service sectors and residences are considered. It has been shown (Evans, 1971; Batty and Mackie, 1972) that for trip models with exponential cost functions the maximization of the likelihood is equivalent to making the mean trip cost of the predicted trips equal to the actual mean trip cost. Thus, β^{W} or actually β' , β^{S}_{k} and λ can be estimated by solving:

$$\sum_{ij} \frac{T_{ij}}{T_{\bullet\bullet}} \stackrel{\sim}{c}_{ij} = \sum_{ij} \frac{T_{ij}^{obs}}{T_{\bullet\bullet}^{obs}} \stackrel{\sim}{c}_{ij}^{obs}$$
(66)

$$\sum_{ijm} \frac{T_{ijm}}{T_{ij^{\bullet}}} c_{ijm} = \sum_{ijm} \frac{T_{ijm}^{obs}}{T_{ij^{\bullet}}^{obs}} c_{ijm}$$
(67)

$$\sum_{ij} \frac{S_{ij}}{S_{**}} c_{ij}^{S} = \sum_{ij} \frac{S_{ij}^{obs}}{S_{**}^{obs}} c_{ij}^{S}$$
(68)

There is a minor problem with this approach; the value of the left hand side of (66) is not known because the composite cost \tilde{c}_{ij}^{obs} is a function of the unknown parameter λ^{obs} . To facilitate the estimation of the three equations it can be assumed that $\tilde{c}_{ij} = \tilde{c}_{ij}^{obs}$.

An alternative approach avoiding this pitfall is to calibrate on the entropy of the transportation matrix. Erlander(1977, 1980), Chon(1982) and Boyce et. al.(1983) have shown that calibrating on the mean trip cost is equivalent to calibrating on the dispersion of the observed travel choices which is defined by the entropy of the system. In that case the three trip deterrence parameters can be estimated from:

$$H_{\beta}^{W} = \sum_{ij} \frac{T_{ij^{*}}}{T_{\bullet \bullet \bullet}} \ln \frac{T_{ij^{*}}}{T_{\bullet \bullet \bullet}} = \frac{T_{ij^{*}}^{\text{obs}}}{T_{\bullet \bullet \bullet}^{\text{obs}}} \ln \frac{T_{ij^{*}}^{\text{obs}}}{T_{\bullet \bullet \bullet}^{\text{obs}}} = H_{\beta}^{W,\text{obs}}$$
(69)

$$H_{\lambda}^{W} = \sum_{ijm} \frac{T_{ijm}}{T_{\bullet \bullet \bullet}} \ln \frac{T_{ijm}}{T_{\bullet \bullet \bullet}} = \frac{T_{ijm}^{obs}}{T_{\bullet \bullet \bullet}^{obs}} \ln \frac{T_{ijm}^{obs}}{T_{\bullet \bullet \bullet}^{obs}} = H_{\lambda}^{W,obs}$$
(70)

$$H_{\beta}^{S} = \sum_{ij} \frac{S_{ij}}{S_{\bullet \bullet}} \ln \frac{S_{ij}}{S_{\bullet \bullet}} = \frac{S_{ij}^{obs}}{S_{\bullet \bullet}^{obs}} \ln \frac{S_{ij}^{obs}}{S_{\bullet \bullet}^{obs}} = H_{\beta}^{S,obs}$$
(71)

Rather than apply the commonly used Newton-Raphson method to solve (69), (70) and (71) simultaneously, a heuristic approach was followed. It was found that β_k^S is not sensitive to changes in the work trip parameters, something probably attributed to the fact that the shopping trips matrix is more aggregated than the one of the work trips. Thus, the shopping trips deterrence parameter is calibrated independently of the work trip parameters, while β^W and λ are estimated through a direct search procedure. The steps of the search procedure were the following:

Step 1. Set number of iterations n=1 and assume an initial value of $^1\lambda$.

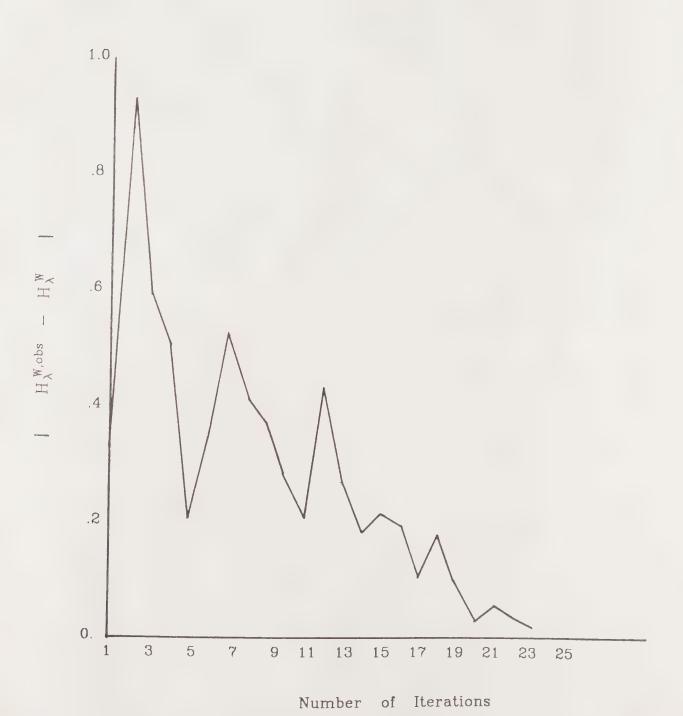
- Step 2. Use $^n\lambda$ to estimate $^{\sim}_{ij}$. Determine β^{W} by calibrating on the distribution entropy $H^{W,obs}_{\beta}$.
- Step 3. Compute modal trip matrix T_{ijm} and the modal entropy H^W_{λ} . Compare it with the observed entropy $H^{W,obs}_{\lambda}$. If they are equal the procedure terminates; otherwise go to step 4.
- Step 4. Estimate ⁿ⁺¹λ through a three-point interpolation procedure similar to that first suggested by Hyman (1969) and later extended by I. Williams (1977). Increase the number of iterations n by one and go to step 2.

As shown in Figure 4 this procedure was relatively efficient; it took twenty three iterations to reach an acceptable level of convergence. The calibration of β^{W} (Step 2) was more rapid; by explicitly taking advantage of the fact that $\beta' \leq \lambda$ (H. Williams, 1977) convergence was achieved in seven iterations on the average.

Because the shopping trip data were not categorized by sector the same β_k^S parameter was used for both Retail Trade and Services. An alternative procedure would have been to calibrate on trip ends using the gradient of the likelihood function as the criterion of goodness-of-fit (Putman and Ducca, 1978).

The second stage of the calibration procedure consisted of modifying the values for the trip parameters β^W and β^S_k so that the model reproduces the observed 1980 distribution of housing and local services employment. Using 1975 as base year, the actual location of basic employment in 1980 and the calibrated modal split parameter λ , the model was estimated for several sets of values of β^W and β^S_k . The set of trip distribution parameters calibrated on stage one were used to initialize the iteractive process. The goodness-of-fit criterion used was the \mathbb{R}^2 defined for housing as:

$$R^{2} = 1 - \frac{\sum_{i} (H_{i} - H_{i}^{obs})^{2}}{\sum_{i} (H_{i}^{obs} - \overline{H}_{i}^{obs})}$$
(72)



and in a similar way for the service sectors.

Finally in the last stage of the calibration the parameters on the agglomeration function were computed. Using the already obtained parameters $\beta^{\mathbb{W}}$, λ and $\beta^{\mathbb{S}}_k$ and 1975 as base year, the model was estimated for several values of α^n . Through a sequential search procedure the values of α^n which maximized the correspondence, as measured by the \mathbb{R}^2 criterion, between the predicted and actual locational patterns of the basic sectors were selected.

Although the calibration methodology was heuristic, in the sense that it does not simultaneously estimate all the parameters, we believe that only marginally improved results would be obtained through a more vigorous approach. Through extensive testing it was determined that some of the parameters are mutually independent; for example both the α^n and the β^S_k parameters were found to be relatively insensitive to small changes in the travel to work parameters.

An interesting aspect of the calibration methodology is that it follows a bottom up approach. The transportation components of the model are calibrated before the locational part. This is unlike the approach used for most land use studies where a top down methodology results into calibrating first the parameters related to the locational decisions and then those describing travel behavior.

The results of the calibrations are summarized in Table 1. The location of housing and employment obtained by using the calibrated model to forecast 1980 values are compared with the actual 1980 data. The goodness-of-fit of both the total values for housing/employment as well as the changes that occurred over the period 1975-1980 are measured.

Overall the statistics depict a fairly good fit. The R² for total housing and for

Table 1. Results of the Calibration

Goodness of fit of predictions with actual 1980 data

| | R |
|------------------------|-----|
| Housing Units | |
| Total | .90 |
| 1975-1980 Change | .74 |
| Total Employment | |
| All Sectors | .89 |
| Manufacturing | .90 |
| Transportation, FIRE | .84 |
| Retail Trade | .91 |
| Services | .86 |
| 1975-1980 Empl. Change | |
| All Sectors | .78 |
| Manufacturing | .75 |
| Transportation, FIRE | .64 |
| Retail Trade | .82 |
| Services | .76 |
| Trips to Work (1975) | |
| Total | .79 |
| Auto | .81 |
| Transit | .69 |

each of the employment sectors are all between .85 and .91. The figures for total employment and housing .89 and .90 respectively indicate an almost perfect fit. These high values also arise from the fact that the model allocates only the total regional change, which for the 1975-1980 period does not exceed the 20% of the already in place housing/employment.

The fit of the model when comparing the forecasts with the actual 1975-1980 change was also acceptable. The R² for housing and employment drop to .74 and .78. There is a wide variation in the fit of the different sectors. Retail Trade is the sector with best fit (.89) while Transportation, FIRE exhibits the worst (.64). The fit of the basic sectors is on the average less successfull then that of the retail/service sectors. This may be attributed to several factors: it is possible that the zonal agglomeration functions were not defined or calibrated correctly, or that some of the factors influencing locational decisions of basic industries were ignored. Additionally, some errors are introduced from the way the basic sectors are defined; the sector of Transportation, FIRE includes employment groups which do not have the same locational characteristics.

An interesting aspect revealed by the goodness of fit statistics is that the model is more successfull in predicting total employment rather than housing. Housing which was not disaggregated either by ownership type or quality characteristics - price range or age and number of units in structure - has a fit which is not as good as the one for total employment. These results indicate that extensions in the model should be in the area of disaggregating housing by type and introducing supply equations linking supply and demand. The fit of total employment is also superior of that of most of the individual sectors, a sign that the model captures the aggregate locational patterns.

When the results obtained at the 107 zone level are collapsed into the nine

counties and compared with the actual county changes the fit of the model improves noticeably. For all the sectors the R² is higher than .85. The improvement is more dramatic for the basic sectors and is undoubtfully due to the existence of the spatial interaction constraints which relate at the county level the size of employment among to different sectors.

The same improvement is also noticed when comparing the predicted and observed employment for twenty large industrial centers, like San Francisco, San Jose, Oakland-Berkeley, Walnut Creek-Concord, Santa Rosa etc. This reaffirms our belief that some of the problems are due to the way the zonal agglomeration function is specified.

Only the fit of the travel to work trips is reported since there was not a detailed trip table for shopping trips. The fit of the work trips is not as good as expected. In aggregates transport studies R^2 in the range of .85 - .95 are not uncommon. However, it should be kept in mind that the model was calibrated to reproduce the 1980 distribution of housing/employment and not necessarily the trip table for 1975. The R^2 's obtained after the first stage of the calibration process were close to .90. Finally, the generalized costs and values of time were taken from another study and might not be the appropriate ones for the proposed model.

Use of the Model for Policy Simulations

A major reason for undertaking the effort of building a new land use information system for the San Francisco Area was the need to provide a tool that would permit the testing of alternative public policies. Some of the policies which can be addressed by the model are:

- What is the impact of changes in local policies regarding land development.
- What is the impact of accelerated shifts in regional employment from Manufacturing to Research and Development, Finance and Services industries.
- What is the impact of new investments in the transit system, like the proposed extension of BART, on the location of housing and employment.

The first issue is related to the development policies of the different cities and counties of the Bay Area. A community's development policies include general and specific plans and other programs to either encourage or discourage development activity in an area. Local zoning regulations for the type and density of new developments, capital improvement schedules and building permit allocation are some of the methods used to manage the rate of growth.

The different land use policies of local government can be used to define the supply of land available for accomodating future households and employment activities. Their impact can be simulated by the proposed model because land constraints are explicitly considered. For example, a policy imposing lower densities for new residential activities can be directly translated into number of acres available for development or potential housing units and its citywide and regiowide impacts on housing location be tested. More important, because the model addresses the issues of housing supply and employment location in a systematic fashion, the impact of the policy restricting housing growth on the number of jobs attracted in the affected areas can be simulated.

The second issue, change in the sectoral composition of employment, can have profound effects in the character of the Bay Area. The new industries have locational patterns and labor force requirements which are in several aspects different than those of traditional manufacturing. They are characterized by an increasing emphasis on decentralization and product specialization, reliance on the availability of a well educated labor force and diminishing requirements for large initial capital outlays. All of these might result into considerable shifts in population and employment. Employment in areas which traditionally have been considered to be 'dormitory towns' might suddenly swell because of the rapid increase in the number of self employed individuals and the formation of many small companies with specialized products, computer software for example.

The emerging tendency in some industries of locating the office close to the homes of the employees might lead to a substantial divergence in the growth rates of the current CBDs and the peripheral areas; eventually, these will have important repercussions on the transport network utilization. Traffic volume on the CBD bound highways and transit systems might remain steady or even decline. In the peripheral areas --Central Contra Costa, Solano and Sonoma counties --, where auto is often the only available mode of transportation, the congestion in the highways might lead to chaotic situations.

POLIS has the capability to simulate the impact that the changes in the composition of regional employment will have at the local level and can assist in the evaluation of the various policy alternatives. Employment is disaggreggated in 5 sectors and can be further disaggreggated should the need for a more detailed analysis arises. Transportation is integrated in the framework and trip flows matrixes for different growth scenarios can be computed and policy implica-

tions be derived.

Finally the last issue, evaluation of investments in transportation infrastructure, is the issue for which the majority of the land use models have traditionally been designed. Investments in the transportation infrastructure alter the accessibility of certain areas which in turn induce locational shifts. Additionally, as noted above, the change in the locational patterns might lead to an increased demand for transport in areas where the infrastructure is inadequate. It must be aknowledged that, because of the relatively high level of aggregation of the transport network, the model does not lend itself for the simulation of minor transportation investments, eg. highway interchanges, but only of major additions to the system. The proposed extensions of the BART system and the provision of improved transit services in the high growth areas are examples of investments whose impacts could be tested by POLIS.

Further Extensions

There are clearly several areas where the model proposed in this study could be extended. The list of the potential extensions and further work can be aggregated into two major groups; the first group includes extensions to provide the model with an improved simulation of the housing markets and individual behavior, while the second group consists of procedures to calibrate the model using more efficient algorithms and more complete data sets.

The goodness-of-fit statistics point out to the area where the major thrust of any extensions to POLIS must be focused. The simulation of the housing sector is not particularly successfull. Housing and household formation is assumed to be driven by the availability of land and the residential attractiveness factor. There is no disaggreggation by housing type or form of ownership and housing

prices are not considered. As a first step to relieve some of these problems, two types of housing, single family and multifamily, and two kinds of housing tenure, own and rent, must be introduced. The latter is particularly important because the characteristics of the households that own their residences is completely different than those that rent.

As a second step, equations linking housing supply to housing demand must be designed. As formulated now, the model can not capture any relative changes in housing prices; it assumes that base year housing prices differentials will remain stable throughout the forecasting period. Housing supply equations similar to those used in the Chicago Area Transportation-Land Use Analysis System (CATLAS) (Anas, 1983) or the NBER model can be specified. Land availability may continue to act as constraint to the provision of housing or be incorporated in the supply equations. In this case prices of new housing will be driven to infinity when the land supply has been exhausted.

Household formation is another area where additional work is needed. Right now household sizes and number of workers per household are estimated exogenously and are not tied to incomes. Introducing equations bringing together the size of the household, the number of workers, the type of tenure and the housing prices/rents will make the model sensitive to changes in the household formation rates and capable of answering questions with respect the demand for housing when the trend twords a smaller household size is reversed.

The capability of the model to simulate the locational decisions of the different employment groups may be enhanced by redefining the zonal agglomeration economies function. Despite the introduction of the exponent in the f_i^n functions, zonal agglomerations are almost linear and can not therefore capture some of the dynamic effects that take place through time. It may be worthwhile

to analyse the location of individual firms using random utility and replace the agglomeration function in the objective function by a component similar in structure to the one used for simulating the selection of residences by individuals.

There is also a need to disaggregate economic activity into more uniform sectors. Although the four sectors categorization, used in the current implementation of POLIS, is an improvement over the traditional basic/nonbasic separation of employment, important locational decisions are probably still hidden. Sectors with different growth rates and structural characteristics are lumped together. For example, the sector of Manufacturing represents Food Processing, a stagnant if not declining industry, and Electronics, a sector exhibiting a very rapid growth. Another factor to be considered when redefining the sectors is the impact of technology on the locational characteristics. The increasing importance of telecommunications and the change of the economy from one driven by the exchange of material goods to one where information is the main commodity will not affect all the industries in the same way.

The adoption of the different extensions proposed above entails several technical difficulties which must not be underemphasized. The introduction of nonlinearities --housing and agglomeration economies -- must be limited to the objective function. Incorporating nonlinear constraints will render the model insolvable since there are no algorithms to handle large problems with nonlinear objective function and constraints. Additionally, there is a tradeoff between the number of sectors and housing types considered and the difficulties encountered during calibration and solution time.

The calibration procedure used in the current implementation of POLIS is a heuristic one and probably fails to capture some of the interdependencies

existing among the various parameters. Theoretical integrity was sacrificed to the necessity of calibrating the model within a short time and a limited data set. A more complete calibration procedure based on the Newton-Raphson method must be designed. Also, additional work must be carried out on the tradeoffs between a simultaneous calibration of all the parameters and a calibration in stages.

Finally the model must be calibrated again as soon as more complete data become available. The data used in the current calibration were from different time periods and different sources. The Bureau of the Census recently released the Urban Transportation Planning Package (UTPP) from the 1980 Census which contains important information on the travel patterns. It provides detailed commuting patterns by mode and census tract of residence and employment in addition to new socioeconomic characteristics. Use of this data set in lieu of the 1975 trip patterns will result in a better simulation of the trip flows matrix and residence location.

Summary

This study presented the structure and the empirical estimation of POLIS, the land use information and transportation model for the San Francisco region. The proposed model advances the state of art of land use modeling by overcoming some of the limitations associated with the Lowry framework. It is aggregate but it results in expressions which are consistent with individual behavior, it considers the location of basic/nonbasic sectors and residences in an integrated fashion, and can easily assess the impact of planning constraints and other development policies.

In the last seven years the field of operational urban models has been in a stagnant state. Disillusion with the early applications of the Lowry model has led most public agencies to abandon efforts to build large scale models. To our knowledge, POLIS is the only comprehensive land use - transportation model implemented for a metropolitan area of the United States in the last five years. The calibration of the model in the Bay Area points out that meaningful models can be calibrated even with limited data availability and limited resources. Finally, the successful use of POLIS for producing long range subregional forecasts for the Bay Area is an indication that large models can be useful for planning purposes if cast within the appropriate framework.



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Guide to Notation

| E_{j} | Total eployment in zone j |
|------------------------|---|
| E_i^B | Total basic employment in zone i |
| Ε̈́p | Regional employment in sector n to be allocated among the zones |
| E_j^n | Employment in sector n in zone j |
| E_j^{no} | Employment at the base year in sector n in zone j |
| Ħ. | Housing units to be allocated among the zones |
| H_{i} | Housing units (households) in zone i |
| $\bar{\mathbb{L}}_{j}$ | Total available land in zone j |
| LS | Locational surplus |
| P_{ij} | Probability of residing at i and working at j |
| P_{ijmk} | Probability of residing at i, working at j, commuting by mode m and shopping at location k |
| $P_{m ij}$ | Conditional probability of commuting by mode m, given that place of residence and work are respectively at zones i and j |
| $P_{\mathbf{k} ijm}$ | Conditional probability of shopping at k, given that place of residence is at i, place of work is at j and mode m is used for commuting |
| S_{ij}^{k} | Number of shopping trips from zone i to service activities of sector k in zone j; or, volume of sales of goods of type k from establishments at zone j to residents of zone i |
| T_{ij} | Number of work trips from zone i to zone j |
| T_{ijm} | Number of work trips from zone i to zone j by mode m |
| U_{ij} | Net utility of living at i and working at j |
| U_{ijmk} | Net utility of living at zone i, working at zone j, travelling to work by mode m and shopping at k. |
| $U_{m ij}$ | Net utility of commuting by mode m given that place of residence and work are respectively at zones i and j |
| $U_{k mij}$ | Net utility of shopping at k given that place of residence is at i, place of work is at j and mode m is used for the travel to work |

| $\overline{ m V}_{ m i}$ | Vacant residential land in zone i |
|---|---|
| \mathbf{W}_{i} | Attractiveness factor for residential activity in zone i; represents trip end utility |
| W_j^k | Attractiveness factor for service sector k in zone j; represents trip end utility |
| Y_j | Total non residential developed land in zone j |
| c_{ijm} | Cost of traveling by mode m from i to j |
| $\stackrel{\sim}{\mathrm{e}}_{\mathrm{ij}}$ | Composite travel cost between i and j |
| $c_{i\mathbf{k}}^{\mathbf{S}}$ | Travel cost from i to shopping activities at location k |
| e_i^k | Mean expenditure of households of zone i for goods of sector k |
| f_i^n | Zonal agglomeration index for zone i and basic sector n |
| g_j^k | Accessibility index for zone j and shopping activity k |
| q_j | Housing affordability index for zone i; ratio of median household income and median housing price at zone i |
| u_{j^*} | Mean surplus associated with employment in zone j |
| u_k^S | Trip end utility of shopping at zone k |
| u_m^{W} | Utility associated with commuting by mode m |
| u_i^{W} | Utility of living at zone i |
| u_{ij}^{W} | Comparative advantage (disadvantage) of commuting from i to j |
| $u_{i^{\bullet}}^{\mathbf{S}}$ | Comparative advantage of shopping from zone i |
| ΔE_j^n | Number of new jobs of sector n in zone j |
| $\Delta \bar{\mathbb{E}}_{j,lb}^{n}$ | Lower bound for the jobs of sector n to be allocated in zone j |
| $\Delta \overline{E}_{j,ub}^{n}$ | Upper bound for the jobs of sector n to be allocated in zone j |
| $\Delta H_{\rm i}$ | New housing units in zone i |
| α^n | Exponent for the agglomeration index of sector n |
| | |

RW

λ

 μ_i^k

 $\nu_{\rm j}$

£n

 $\rho_{\rm i}$

 $\varphi_{\rm c}^{\rm n}$

| β^{W} | Deterrence parameter for the distribution of commuting trips; converts utility into monetary units compatible with the trip cost |
|---------------------------------|--|
| $oldsymbol{eta}^{\mathbb{S}}$ | Deterrence parameter for the distribution of shopping trips; converts utility into monetary units compatible with the trip cost |
| β' | Deterrence parameter equal to $\frac{\beta^{W} \lambda}{\beta^{W} + \lambda}$ |
| $\gamma_{ m j}$ | Dual variable of constraint (20) |
| $oldsymbol{\delta}_{	ext{i}}$ | Dual variable of constraint (19) |
| $oldsymbol{\eta}_{ m i}^{ f k}$ | Dual variable of constraint (43) |
| $\vartheta_{\rm j}$ | Dual variable of constraint (42) |
| $\kappa_{ m i}$ | Dual variable of constraint (41) |

Dual variable of constraint (44)

Dual variable of constraint (45)

Dual variable of constraint (47)

Dual variable of constraint (46)

Dual variable of constraint (48)

Dual variable of constraint (49)

Deterrence parameter for the modal split of the journey to work trips; converts utility into monetary units compatible with the trip cost



APPENDIX A

Input Data Needed for Using POLIS

I. General

- . number of zones
- , number of modes
- . number of employment sectors
- . housing units to be allocated
- . regional employment to be allocated

II. Calibration parameters

- . deterrence parameters for work trips (distribution and modal split)
- . deterrence parameter for shopping trips
- . agglomeration function exponent

III. For every zone

- . base year population, housing units, employment (by sector)
- . base year land use conditions in acres
 - land use utilization by type (residential, basic + nonbasic)
 - vacant land by type (residential, basic + nonbasic)
- . densities
 - residential (housing units per acre)
 - employment by sector (employees per acre)
- . attractiveness factors
 - residential
 - nonbasic sectors
- . zonal agglomeration functions for basic sectors
- . workers per household, household size

IV. Transportation

- . peak hour trip cost matrix for every mode (auto, transit)
- . non peak hour trip cost matrix for auto

Input Data for a Sample Zone

Zone 20 Millbrae/Burlingame

1985 Allocation

I. Base year (1980) input data

. Households: 24745

. Total Employment: 44819

Agriculture: 396 Manufacturing: 5309 Transp., FIRE: 23116

Retail: 5598 Services: 10400

. Developed residential land: 5068 acres

. Developed basic + nonbasic land : 1439 acres

II. 1985 Densities and vacant land

. Densities

Residential: 4.92 units/acre

Agriculture: 2.3 employees/acre

Manufacturing: 30.7 "
Transp., FIRE: 39.5 "
Retail: 36.3 "
Services: 32.8 "

. Vacant land (recycled land included)

Residential: 27.6 acres
Basic + nonbasic: 325.1 acres

III. Attractiveness and agglomeration factors

. Attractiveness factors (normalized)

Residential: .037

Retail Employment: .502

Service Employment: .810

. Zonal Agglomeration factors

Manufacturing: .053 Transp., FIRE: .324

IV. Other input data

. Household size : 2.41 persons

. Workers per household: 1.25

. % Population in group quarters: 11.3%



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